pcTVI: Parallel MDP Solver

Using a Decomposition Into Independent Chains

Jaël Champagne Gareau
Éric Beaudry
Vladimir Makarenkov

Computer Science Department
Université du Québec à Montréal

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Outline

Automated planning & scheduling

- **Automated planning & scheduling** is a branch of Artificial Intelligence.
- Its objective is to find **plans** allowing **agents** to reach **goals**.
- Some planning problems are **probabilistic** (i.e., there are **uncertainties**) :
  - endogenous uncertainties (i.e., due to the agent) ;
  - exogeneous uncertainties (i.e., due to the environment).
- **Markov Decision Processes** (MDPs) are often used to model these problems of decision-making under uncertainty.

Objective of this research

- Most interesting real-world MDP problems require a large number of state variables.
- Curse of dimensionality : Number of states is exponential in the number of state variables.
- Often we are limited in time to find the problem's solution.
- Therefore, we need to find ways to accelerate MDP computations.

Mathematical MDP representation

- There exists many variants of MDPs. The most common are :
  - **Finite-horizon MDP** ;
  - **Infinite-horizon Discounted MDP** ;
  - **Stochastic Shortest-Path MDP** (SSP-MDP).
- We focus on SSP-MDPs, since they are more general.

Stochastic Shortest-Path MDP

An **SSP-MDP** is a tuple (S, A, T, C, G) where :

- S is the finite set of **states** ;
- A is the finite set of **actions** that the agent can execute ;
- $T: S \times A \times S \to [0, 1]$ is the **transition function**, where $T(s, a, s')$ gives the probability that the agent reaches state s' if it executes action a at state s ;
- $C: S \times A \times S \to \mathbb{R}^+$ is the **cost function** where $C(s, a, s')$ gives the cost an agent must pay if it reaches state s' when executing action a at state s ;
- $G \subseteq S$ is the set of **goal states**.

Classical algorithms

Policy

A (Markovian and stationnary) **policy** is a function $\pi : S \rightarrow A$ that returns, for every state, the action an agent should execute.

Value function

A **value function** (associated to a policy π) is a function $V^{\pi} : S \rightarrow \mathbb{R}$ that maps each state s to the expected total cost of an agent starting at s that executes the actions given by π until reaching a goal.

Classical algorithms

- Policy Iteration (PI) [1]
- Value Iteration (VI) [2]

1. Howard, R. A. (1960). Dynamic Programming and Markov Processes. John Wiley.
2. Bellman, R. (1957). Dynamic Programming. Prentice Hall.

Modern approaches

Heuristic search

- These methods assume that we have two additionnal elements :
  1. an initial state known a priori ;
  2. a **heuristic function** $h \colon S \to \mathbb{R}$ estimating the expected cost to reach a goal.
- Common MDP heuristic search algorithms :
  - **LAO*** (ILAO*, RLAO*, BLAO*, etc.), **LRTDP** (BRTDP, FRTDP, etc.).

Prioritized methods

- The order of states' value update drastically influence convergence time.
- E.g., can range from $\mathcal{O}(n)$ to $\mathcal{O}(n^2)$ state updates.
- **Prioritized VI** (PVI) : a priority is assigned to every state.
- There's many variants with different priority function :
  - Prioritize states close to a goal (for a more efficient back-propagation of states' value) ;
  - Prioritize states with a large residual (farthest from convergence) ;
- Newer methods partition the MDP and assign priority to them rather than states.
- E.g., **Topological Value Iteration** (TVI), FTVI, P3VI, etc.

Exploiting modern computer architectures

- Another way of improving speed is to consider the architecture of modern computers, e.g. :
  - Cache memory hierarchy [3] ;
  - GPU implementation.
  - Data-level parallelism (i.e., SIMD operations) ;
  - Thread-level **Parallelism** ;
- Many considerable speedups have been obtained in other domains.
- E.g., in ML, many researches have recently provided some efficient implementations of ML techniques (specialized data structures, specialized CPU datatype (bfloat), consideration of cache, parallel decomposition, etc.).
- In MDP planning, no such elements have ever been considered.
- Our goal with this research is to propose a parallel MDP solver based on TVI.

3. Champagne Gareau, J., Beaudry, É., & Makarenkov, V. (2022). Cache-Efficient Memory Representation of Markov Decision Processes. Proceedings of the Canadian Conference on Artificial Intelligence. https ://caiac.pubpub.org/pub/pq25qiqh

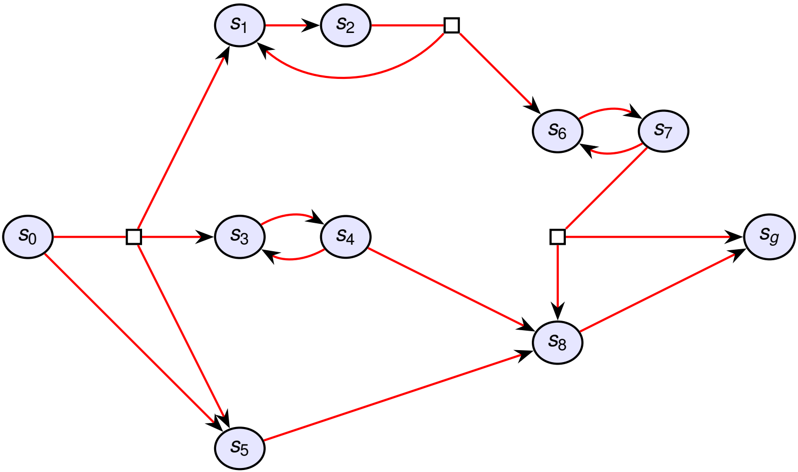
Existing parallel MDP algorithms

- P3VI : Partitionned, Prioritized, Parallel Value Iteration
  - Partitions the state-space into multiple sub-parts ;
  - Assign a priority to each part ;
  - Solve multiple parts in parallel in the order given by the priorities.
  - Disadvantages :
    - The partitionning is done on a case-by-case basis depending on the planning domain ;
    - Communication of the state values between the solving threads incurs an overhead on the computational time.
- Parallel CECA (Cache-Efficient with Clustering and Annealing)
  - CECA partitions the state-space and solves the parts one-by-one according to a simulated-annealing schedule.
  - Parallel CECA solves multiple clusters in parallel.
  - Disadvantages :
    - The final algorithm is more complex to understand/implement than other MDP algorithms ;
    - The performance improvement due to the parallelization is underwhelming (factor 2.59 speedup on a 10 cores CPU).

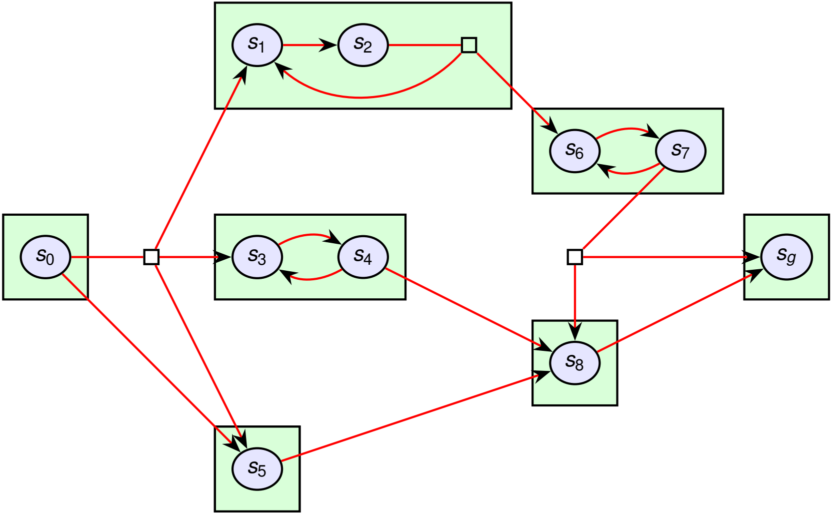
Parallel-Chained Topological Value Iteration

- Based on Topological Value Iteration ;
  - Considers the graph corresponding to the topological structure of the MDP (or, equivalently, the all-outcome determinization of the MDP) ;
  - Uses Tarjan's algorithm to decompose the graph into strongly-connected components (SCCs) ;
  - If we consider the SCCs in reverse topological order, they can be solved using a single sweep over each of them.
- Instead of choosing SCCs to solve in parallel randomly or with a priority metric, we use instead the dependencies between the SCCs.
- SCC's with no common dependencies can be computed in parallel.
- To find the dependencies : we can do a backward breadth-first search (from the goal state) in the graph condensation of the MDP (the graph containing the SCCs of the MDP structure) to find chains of independent SCCs.
- During runtime, a new parallel task is created everytime an SCC's dependencies have all been computed.

Example of pcTVI execution



Example of pcTVI execution



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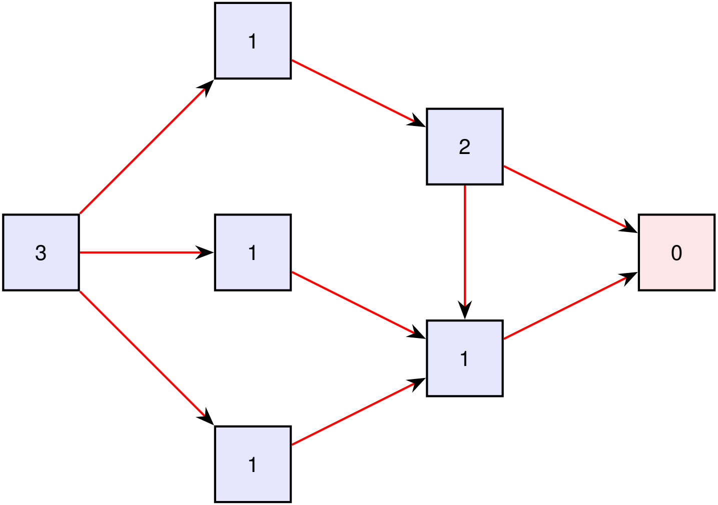

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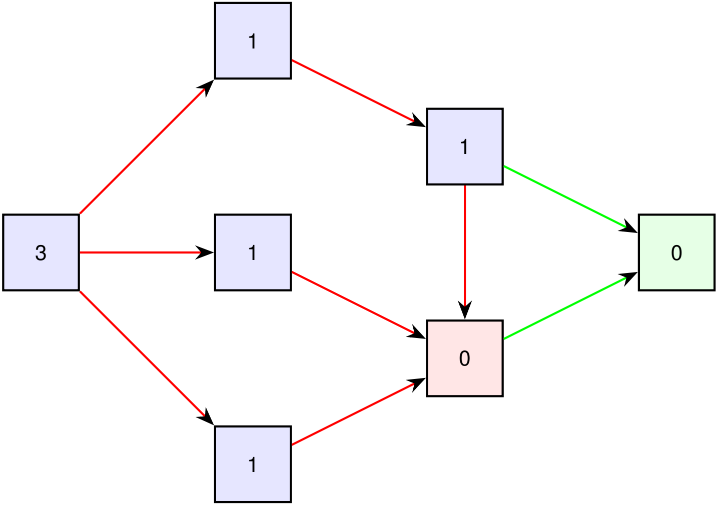

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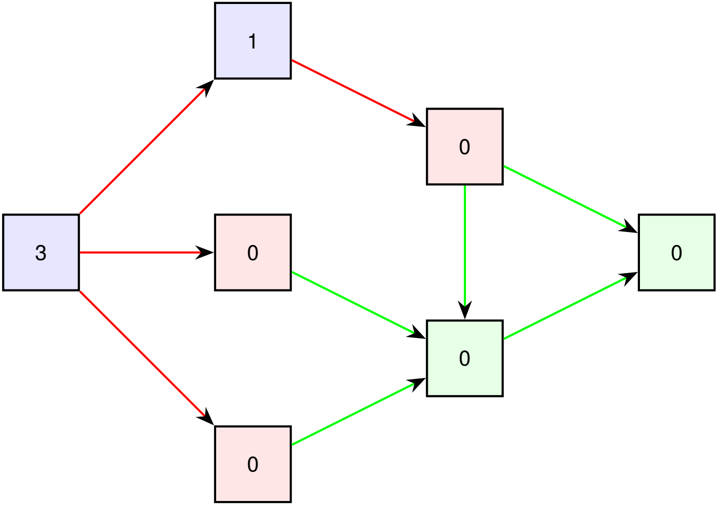

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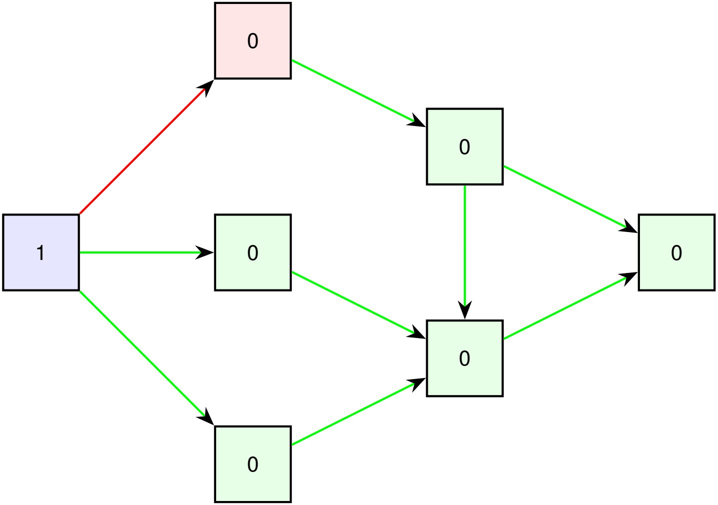

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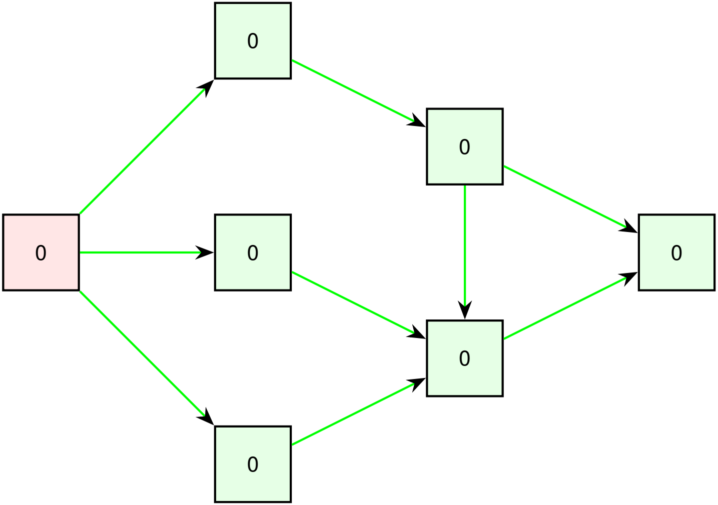


Algorithm Parallel-Chained Topological Value Iteration

```
procedure pcTVI(M : MDP, t : Number of threads)
    ▷ Find the SCCs of M
    G ← Graph(M)                          ▷ G implicitly shares the same data structures as M
    SCCs ← Tarjan(G)                      ▷ SCCs are found in reverse topological order

    ▷ Solve in parallel independent SCCs
    G_c ← GraphCondensation(G, SCCs)
    Pool ← CreateThreadPool(t)            ▷ Create t threads
    V ← NewValueFunction()                ▷ Arbitrarily initialized ; Shared by all threads
    Q ← CreateQueue()                     ▷ Shared by all threads
    Insert(Q, Head(SCCs))                 ▷ The goal SCC is inserted in the queue
    while NotEmpty(Q) do                  ▷ Only one thread runs this loop
        scc ← ExtractNextItem(Q)
        for all neighbor ∈ Neighbors(scc) do
            Decrement NumIncomingNeighbors(neighbor)
            if NumIncomingNeighbors(neighbor) = 0 then
                AssignTaskToAvailableThread(Pool, PartialVI(M, V, scc))
                Push(Q, scc)              ▷ Neighbors of scc are ready to be considered next

    return GreedyPolicy(V)
```

Chained-MDP domain

- There was no standard probabilistic planning domain in the literature suitable to benchmark a parallel MDP solver.
- We thus propose a new parametric MDP solver, called **Chained-MDP**.
- The parameters are :
  - k : the number of independent chains $c_1, c_2, \ldots, c_k$;
  - n_{scc} : the number of SCCs $scc_{i,1}, scc_{i,2}, \ldots, scc_{i,n_{scc}}$ in every chain c_i ;
  - n_{sps} : the number of states per SCC ;
  - n_a : the number of applicable actions per state ;
  - n_e : the number of probabilistic effects per action.
- Successors of a state s in $scc_{i,j}$ can be any state in $scc_{i,j}$ or in $scc_{i+1,j}$ (or, if the latter does not exist, it can be the goal state).

Chained-MDP instance example

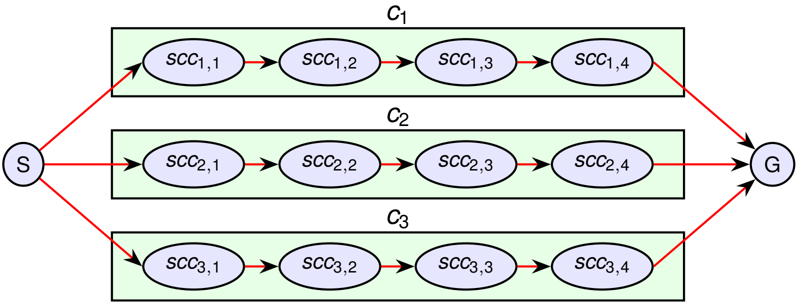
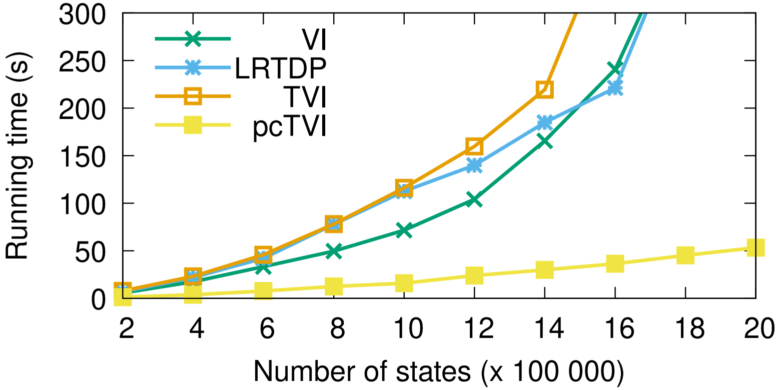


Figure – A chained-MDP instance where $n_c = 3$ and $n_{scc} = 4$. Each ellipse represents a SCC.

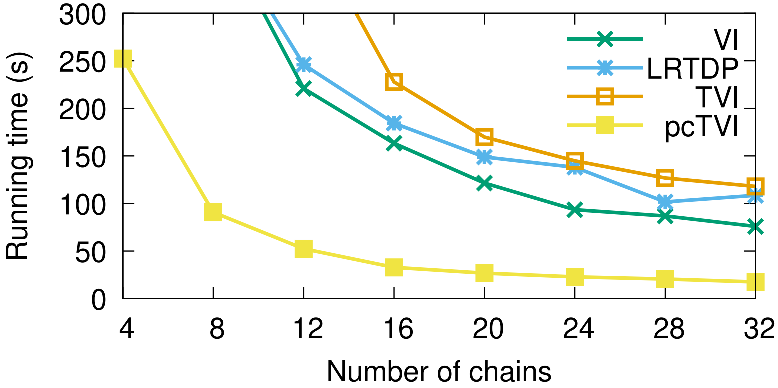
Methodology

- We compare the performance of pcTVI to the performance of :
  - VI (the asynchroneous round-robin variant) ;
  - LRTDP (with the admissible and domain independent h_{min} heuristic) ;
  - TVI.
- We implemented the proposed algorithms in C++.
- We used the GNU g++ compiler (version 11.2) with level 3 optimizations.
- The tests were carried out on a computer equipped with four Intel Xeon E5-2620V4 processors.
- Each of these processors have 8 cores (at 2.1 GHz), for a total of 32 cores.
- The planner never used more than 2 GB, even for the largest domain instances so memory usage of our proposed algorithm was not an issue.
- For every test instance, we measured the running time of each algorithms carried out until convergence to an ϵ-optimal value-function (we used $\epsilon = 10^{-6}$).
- For each tested parameter configurations of the parallel-chained MDP domain, we randomly generated 15 instances.
- To minimize random factors, we report the average values of the obtained results.

Chained-MDP with varying number of states and a fixed 32 chains



Chained-MDP with fixed 1M states and varying number of chains



Conclusion

- Finding an ϵ-optimal policy of an MDP can take an unreasonable amount of time due to the curse of dimensionality.
- We proposed a domain-independent way of solving an MDP in parallel.
- We also proposed a new parametric planning domain, suitable to model any sitiation where different strategies (i.e., a chain) can reach a goal but where, once commited to a strategy, it is not possible to switch to a different one.
- The pcTVI algorithm led to an average speedup of 20, on a 32 cores computer.
- As future work, we plan to :
  - investigate ways of pruning provably suboptimal actions, which would allow more SCCs to be found ;
  - investigate on how to apply the proposed algorithm on the MDPs used in Reinforcement Learning (RL).

Acknowledgments

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